

# Pentaquark in a supersymmetric quark-diquark model

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## Abstract

According to QCD, there exists a broken dynamical supersymmetry between an antiquark and a diquark. This supersymmetry can be used to relate the mass of a pentaquark to the mass of an antibaryon by replacing two antiquarks in an antibaryon by two diquarks to form a pentaquark. Using this technique, we find that the mass of an exotic pentaquark with strangeness plus 1 is greater than 1.74 GeV, or at least 200 MeV larger than that of the reported  $\Theta^+$  pentaquark. Furthermore, there is no reason for the pentaquark to be narrow; on the contrary, it is expected to be so broad that it will be difficult to observe.

Several groups [1–4] have claimed evidence for a baryon called  $\Theta^+(1540)$  with positive strangeness. Additional observations were reported at a recent conference on Quarks and Nuclear Physics [5]. A baryon with strangeness  $S = 1$  cannot consist of three quarks, but must contain at least four quarks and an antiquark, in other words, must be a pentaquark or a still more complicated object. Other people [5, 6] have cast doubt on whether the pentaquark has in fact been established.

Of course it is an experimental question whether the  $\Theta^+$  exists or not, but it is still of interest to see what QCD has to say on the subject. So far, to our knowledge, calculations of the pentaquark mass have been made in lattice QCD only in the quenched approximation. We mention two such calculations, one by Sasaki [7], who finds the lowest-mass baryon with strangeness  $S = 1$  to be more than 200 MeV higher than the reported mass of the  $\Theta^+$  (he says not to take the predicted mass too seriously). On the other hand, Csikor et al [8] calculate a mass that they say is consistent with that of the  $\Theta^+$ . In the absence of a unique value of the  $\Theta^+$  mass from lattice calculations, we use a simple model based on QCD to predict a lower limit to the mass of the  $S = 1$  pentaquark. We use the framework of the constituent quark model.

According to QCD, an approximate dynamical supersymmetry exists between an antiquark and a diquark. The first person to point out a supersymmetry between antiquarks (half-integral spin) and diquarks (integral spin) was Miyazawa [9] in 1966, several years before it was suggested that a supersymmetry might exist between elementary quarks and squarks, between electrons and selectrons, etc. In

fact, we think that Miyazawa was the first person to apply the idea of supersymmetry to particle physics. (Miyazawa did not use either the word “diquark” or the word “supersymmetry” in his paper, but the concepts are there.)

The point is that both a diquark and an antiquark belong to an antitriplet of  $SU(3)$ -color, and to first approximation the interaction of QCD depends only on color. A diquark can also belong to a sextet of color, but a sextet diquark is irrelevant to the supersymmetry we are discussing. In lowest-order QCD perturbation theory, two quarks in a color sextet are subject to a repulsive force, with the consequence that a sextet diquark has a higher mass than an antitriplet diquark. We have no reason to believe that this result will change in full QCD.

Catto and Gürsey [10] were the first to point out that the supersymmetry between antiquark and antitriplet diquark is an approximate consequence of QCD and that the supersymmetry is responsible for the fact that mesons and baryons have Regge trajectories with approximately the same slope. A review of diquarks, including a section on supersymmetry, appeared in a paper in 1993 [11].

We can think of at least three things that break the supersymmetry:

1) A quark and a diquark have different masses. It is easy to take this mass difference into account in approximate fashion by assigning a diquark to have a mass equal to the sum of the masses of the quarks it contains.

2) A quark and a diquark have different sizes. A diquark may be almost the size of a hadron, whereas a current quark is assumed to be pointlike. However, a constituent quark, with its cloud of gluons and quark-antiquark pairs, is certainly not pointlike. We neglect the size difference between a constituent antiquark and a diquark.

3) A quark and a diquark have different spins, and in QCD there are spin-dependent interactions. In the present paper, we adopt a procedure which does not quantitatively take into account the spin dependence of quark and diquark interactions. However, we deduce from the systematics of baryon masses that a spin-zero diquark has a lower mass than a diquark of spin one.

A few years ago, similar ideas were used [12] to predict the masses of exotic mesons (four-quark states) and dibaryons (six-quark states) but not the properties of pentaquarks. It was found that quadriquarks and sexaquarks, with the possible exception of states containing two  $b$  quarks, are highly unbound in the model and probably have decay widths too large to be easily observed.

Here, we do not follow Ref. [12] in detail but use simpler arguments, which we hope are more general, to show that a pentaquark state with properties of the  $\Theta^+$  does not exist in the model. We can choose quark constituent masses such that a ground-state hadron is the sum of the masses of the quarks it contains. Except in the heavy-quark limit, such a procedure requires us to assign different masses to quarks of the same flavor in different hadrons. But if we are interested just in lower limits to certain hadron masses, we can avoid this complication.

First, we consider an example of a meson and a baryon containing only  $u$  and  $d$  quarks, and neglect the mass difference between the  $u$  and  $d$ . The lightest meson

containing these quarks is the pion, while the lightest baryon is the nucleon. If the  $u$  and  $d$  quarks were infinitely heavy, the nucleon mass would be  $3/2$  times the mass of the pion, but of course actually the nucleon mass is considerably larger than  $3/2$  times the pion mass. Part of the reason is that the pion is anomalously light, but the result seems to be more general. For example, if we replace a  $\bar{u}$  antiquark by a light diquark in a  $\bar{K}$  meson, we obtain a strange baryon. The lightest strange baryon is the  $\Lambda$ , and its mass is larger than the  $K$  mass by an amount that is greater than the extra mass of a light constituent quark. The additional mass acquired by a hadron when we replace an antiquark by a diquark is, of course, a breaking of the dynamical supersymmetry. The rule seems to be that *when we replace an antiquark by a diquark in a meson, the resulting baryon has a mass that is greater than the meson mass by an amount that is larger than the mass of the additional quark.*

Likewise, we expect that if we replace two antiquarks in an antibaryon by diquarks, the mass of the resulting baryon (a pentaquark) will be larger than the original antibaryon mass by an amount greater than the two extra quark masses. We can think of no reason why the systematics here should be any different from the meson–baryon case. A model in which a pentaquark consists of two diquarks and an antiquark has previously been considered by Jaffe and Wilczek [13], but not within the framework of diquark-antiquark supersymmetry.

Without further ado, we turn to the problem at hand. Using the fact that the nucleon has a mass of 939 MeV and that it contains three constituent quarks, we choose the mass of a  $u$  or  $d$  quark to be 313 MeV. (We could of course choose a somewhat different constituent quark mass at the expense of including an appropriate binding energy.) The lightest baryon containing a strange quark is the  $\Lambda$ , with a mass of 1115 MeV. If we consider an anti- $\Lambda$  and replace its  $\bar{u}$  and  $\bar{d}$  antiquarks by light diquarks, we add  $2 \times 313 = 626$  MeV to the mass of the  $\Lambda$  (1115 MeV) to obtain a lower limit on the mass of a pentaquark with strangeness  $S = +1$ . We obtain

$$M(\Theta^+) > 1741 \text{ MeV}.$$

This value is at least 200 MeV larger than the reported mass [1–5] of the  $\Theta^+$  baryon.

Because the two spin-zero  $ud$  diquarks in the pentaquark are identical bosons, they cannot have a symmetric spatial wave function. We see this as follows: The wave function of the two diquarks is antisymmetric in color. Therefore, their space wave function must also be antisymmetric so as to make an overall symmetric wave function. One way to accomplish this is by putting the two diquarks in a state with orbital angular momentum one. In this state the energy will probably be a few hundred MeV higher than 1740 MeV, and the parity will be positive.

If, in the model, one of the diquarks has spin one then, by symmetry, that diquark must also have isospin one, in contradiction to the claim that the pentaquark has isospin zero. If both diquarks have spin and isospin one, the total isospin can be zero. From two spin-one diquarks we can construct an antisym-

metric state with spin one. This spin can combine with the spin of the  $\bar{s}$  quark to form total spin  $1/2$  or  $3/2$ , but the systematics of spin-dependent forces tells us that the state with spin  $1/2$  will lie lower. In this case the two diquarks have a symmetric spatial wave function and negative parity, but again we estimate that the energy will be a few hundred MeV higher than 1740 MeV because a spin-one diquark is heavier than a diquark of spin zero. It is not clear whether the state with negative or positive parity will lie lower.

In the model there is nothing to prevent a rapid decay of the pentaquark by color rearrangement into nucleon + kaon. Because the  $Q$ -value of the decay is expected to be well over 300 MeV, the pentaquark should have such a broad width as to make it hard to observe as a resonance, even in the case with orbital angular momentum zero.

We conclude that in a supersymmetric quark-diquark model of hadrons which is motivated by QCD, we do not expect the existence of a state with the reported properties of the  $\Theta^+$  baryon.

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